Research Foundations and Design

VMATH™

Voyager Expanded Learning®
Research and Development

Expanded Learning Voyager®

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Introduction

Many students do not learn effectively from the instruction provided in their regular math classes. They struggle continually with the math concepts, skills, and strategies necessary for success (Cross and Hynes, 1994). These students often fall even further behind as their core program moves on to more difficult material. They score poorly on high-stakes tests, attend summer school, and may even be denied advancement to the next grade level. The NCTM *Principles and Standards for School Mathematics* (PSSM) asserts that "...students with special educational needs must have the opportunities and support they require to attain a substantial understanding of important mathematics." (NCTM, 2000).

Vmath, Voyager Expanded Learning's math intervention series, is a targeted, systematic program that provides students more opportunity and support to learn mathematics. Vmath is informed by Curriculum-Based Measurement and provides daily, direct, systematic instruction in essential math skills needed to reduce achievement gaps and accelerate struggling math students to reach and maintain grade-level performance. Examples in the literature (Campbell, 1995; Griffin, Case, and Siegler, 1994; Knapp et al., 1995; Silver and Stein, 1996) demonstrate that all children, including those who have been traditionally underserved, can learn mathematics when they have access to high-quality instructional programs that support their learning.

Vmath is designed to complement all major math programs by providing an additional 30–40 minutes of daily, targeted concept, skill, and problem-solving development. Each level of Vmath contains 10 individual modules covering the basic strands of elementary mathematics. The content of these modules is aligned with grade-level expectations for the NCTM Content Standards of the PSSM (NCTM, 2000).

Vmath Content Strands

Whole Numbers
Decimals
Number Theory
Fractions
Data Analysis, Statistics, and Probability
Measurement
Geometry
Ratio, Proportion, and Percent
Integers and Rational Numbers
Algebra

Instructional Design

Vmath is designed using widely-accepted principles of effective intervention instruction for struggling students. The direct, systematic instruction in Vmath provides carefully sequenced, specific, and detailed dialogue for every lesson. This form of explicit, highly-structured lesson delivery uses a direct instruction model based on research by Stein, Silbert, and Carnine (1997).

Nine Steps to Designing an Effective Instructional Program

1. Specify long- and short-term objectives.
2. Devise instructional strategies.
3. Determine necessary preskills.
4. Sequence skills.
5. Select a teaching procedure.
6. Design teaching formats.
7. Select examples.
8. Specify practice and review.
9. Design progress-monitoring procedures.

Rosenshine (1983), in a review of research on teacher effectiveness, concluded that instruction that is highly interactive, briskly paced, and clearly presented was related to high rates of student success. He referred to this type of teaching as direct instruction (explicit, teacher-directive practices): however, the term *direct instruction* is generally used to refer to the instructional theory work of Engelmann and Carnine (1991). Research provides consistent support for using an explicit approach to teaching mathematics. Adams and Engelmann (1996) analyzed 34 intervention studies and found this approach to be more successful in 32 of the 34 studies they reviewed. Bottge (2001) asserted that teachers should continue to foster competence in basic skills by providing students explicit instruction. Kroesbergen and Van Luit (2003) reported in a study of 58 research reports that direct instruction was found to be more effective than mediated instruction. All of these studies provide convincing evidence that such pedagogy facilitates the learning process.

Vmath instruction is characterized by a clear four-step lesson format consistent with the tenets of direct instruction: Getting Started; Guided Practice; Independent Practice; and Test Prep and Error Analysis.

Getting Started

Each Vmath lesson begins with a review of preskills in the Getting Started section. "Because students learn by connecting new ideas to prior knowledge, teachers must understand what their students already know." (NCTM, 2000) The first few problems review the prerequisite skills for the lesson or review the component skills of the process being taught later in the lesson. After a review of the preskills, the teacher carefully models the new skill or strategy needed to achieve the lesson objective. In Getting Started, the instruction is explicit with a script for the teacher to follow. No student instructions are written in this section, ensuring that the teacher will use the script to model each step needed by the student to effectively learn the new skill or strategy.

Guided Practice

In the Guided Practice section of each Vmath lesson, strategic integration of content provides a concise and clear procedure for acquiring new knowledge in a "How To" box. The teachers are provided with a specific dialogue that will lead the student in acquiring this knowledge.

The "How To" box summarizes for students the steps needed to master a concept, skill, or strategy. Vmath students are guided by the teacher through the lesson with effective use of prompts and questions that ensure proper student responses (Good, Grouws, and Ebmeier, 1983; Cybriwsky and Schuster, 1990). In Guided Practice, the teacher-directed dialogue is used to scaffold instruction while supporting the student during completion of the problems. Scaffolding provides students with temporary support by structuring the task into manageable chunks. Teachers then assist students in thinking about the chunks. Teachers are guided in the lesson dialogue in the gradual and purposeful removal of the scaffolds (Hall, 2002). During Guided Practice, the teacher monitors student performance and uses standard correction procedures to give the student immediate feedback. Thus, Vmath Guided Practice is a primary means by which the teacher verifies that students can apply the concepts in the next phase of the lesson, Independent Practice.

If necessary, portions or all of the Getting Started and Guided Practice can be repeated and followed by additional practice located in the back of the student book for grades 3–6 and at the end of each lesson for grades 7 and 8.

Independent Practice

Vmath students practice lesson content and previously learned skills from Getting Started and Guided Practice on their own in the Independent Practice section. Students can refer to the "How To" box while completing the problems in the Independent Practice. The teacher monitors and checks each student's work daily. This teaching technique is supported by Smith and Geller (2004), who indicate that feedback is important in effective math instruction for all learners, including learners with disabilities and those at risk of school failure.

Students keep a personal graph of their Independent Practice achievement and progress in the back of their student book. This practice of self-monitoring has been shown by researchers to enhance academic achievement (Lan, Repman, and Chyung, 1998).

Test Prep and Error Analysis

After Independent Practice, Vmath offers students a Test Prep and Error Analysis section that allows them to apply what they have learned in a format similar to a high-stakes assessment situation. Three questions that check the learning in the lesson provide the teacher with information so that effective reteaching can occur, if necessary. Questions written in multiple-choice and short-answer formats are an effective and efficient way to track daily progress. Some of the distracters presented in the multiple-choice problems represent common errors made by students. When these specific wrong answers are selected, the teacher uses the provided correction procedures to correct the error. Error analysis is another of the common attributes that have been identified as positively affecting student learning (Smith and Geller, 2004). This practice of error analysis with reteaching is aligned with the research findings of Good and Grouws (1979) related to improving learning through reteaching.

Essentials of Vmath Instruction

The lessons in Vmath address three essential components of math instruction—concepts, skills, and problem-solving—with a major goal of developing students' computational fluency. A major goal in grades 3–5 is the development of computational fluency with whole numbers (NCTM, 2000). Additionally, Vmath students reach for computational fluency with fractions and decimal numbers by the end of grade 6 or in the beginning of grade 7.

Concepts
Vmath teaches the underlying concepts, or big ideas, needed for understanding mathematics by using visual models and pictorial representations.

Skills
Vmath provides clear step-by-step procedures to guide students through the processes needed to complete basic algorithms successfully.

Problem-Solving
As students master the fundamental math concepts and skills, they also must develop appropriate problem-solving skills and strategies. In Vmath, students use a problem-solving plan, develop problem-solving strategies, and practice problem-solving applications. Explicit instruction in these problem-solving techniques is consistent with results of research studies examined in a meta-analysis of research on instructional strategies (Gersten, Chard, Baker and Lee, *in review*).

Using a Problem-Solving Plan: The problem-solving plan in Vmath incorporates the four phases of the problem-solving plan devised by George Polya in his book *How to Solve It* (Polya, 1957). In the Getting Started section, the teacher guides the students through the process of understanding the problem, making a plan, carrying out the plan, and looking back to review. In the Getting Started and Guided Practice sections, the student is directed to ask questions such as, "What am I trying to find?", "How can I find the answer?", "What operation should I use?", and "Why is my answer correct?"

Developing Problem-Solving Strategies: Vmath teaches four basic strategies that struggling math students should be able to apply in the context of problem-solving. Using the direct, explicit, and systematic format of Vmath, the teacher models the critical-thinking skills and unique procedures in learning and applying each strategy. The strategies include: Using Patterns, Working Backward, Using a Table, and Solving a Simpler Problem.

Practicing Reasonableness, Estimation, and Guess-and-Check Techniques: The final question in the Test Prep and Error Analysis section is an open-ended question that provides the teacher with an opportunity to model the techniques of Checking for Reasonable Answers, Estimation, and Guess-and-Check.

Vmath Assessment

Vmath provides a full range of assessments to enhance instructional decisions, including a Curriculum-Based Measurement component, Pre-Test and Post-Tests, and several opportunities for ongoing assessment. Research suggests that the practice of providing feedback on student performance as well as information related to a student's specific strengths and weaknesses is a combination beneficial to student achievement (Gersten, Chard, Baker and Lee, *in review*).

Curriculum-Based Measurement

The scientific research basis used for developing the Curriculum-Based Measurement System was initiated by a group of educational scientists headed by Dr. Stanley Deno at the University of Minnesota (Deno, 1985). Their research led to a system of measures later called Curriculum-Based Measurement (CBM) that sought to identify reliable and valid ways of assessing students' progress in essential mathematical skills. Research studies have shown that progress monitoring can improve student performance (Baker et al., 2002; Gersten et al., *in review*). CBM has been shown to discriminate between students who achieve typically and those in compensatory programs (Deno, Fuchs, Marston, and Shinn, 2001).

Furthermore, Marston (1989) and Thurber, Shinn and Smolkowski (2002) have shown that having students write answers to grade-level computational problems for a short interval of time is a reliable and valid outcome measure of general mathematics computation for typically achieving students through grade 6. Providing information and instructional suggestions to teachers about student performance was found by Fuchs et al. (1994) to positively affected student performance. Vmath provides both types of information to teachers.

Vmath Benchmark and Progress Monitoring System

One of the most important components of the Vmath intervention system is the Vmath Benchmark and Progress Monitoring System. These assessments in this system include three benchmark assessments, 10 progress monitoring assessments, and a data management system.

For each Vmath level, an assessment prototype is constructed in three steps. First, the computational problem types are identified by examining the computational objectives for that level. Second, these problem types are weighted rationally to reflect their importance to success in mathematics computation. Third, each Vmath Benchmark and Progress Monitoring Assessment is created using the weighted problem types in random orders. Once the prototype is complete, individual items are generated for all assessments.

The result is a mapping of equivalent problems in the same position for each of the 13 assessments. For example, if the first problem in the first row of each assessment is adding two three-digit numbers containing two decimal places with an answer containing four digits, the first problem in each of the 13 assessments will fulfill this same requirement.

Thus, all tests are equivalent forms allowing for measurement of growth over time. These forms are brief screening measures that focus on critical math skills strongly predictive of future growth and success.

CBM results can be used to indicate how well a student is progressing throughout the entire grade level; over time the results become good predictors of grade-level success. One specific finding by Gersten, Chard, Baker, and Lee (*in review*) indicates that providing teachers with specific information on student performance enhances achievement. Vmath CBM as well as the Pre-Tests and Post-Tests provide this type of feedback to teachers.

Pre-Tests and Post-Tests

While Vmath Benchmark and Progress Monitoring Assessments assess computational skills over the entire grade level, Pre-Tests and Post-Tests focus only on mastery of the specific content taught in each module. Pre-Tests determine students' prior knowledge of content and help teachers establish instructional priorities over that specific content. Post-Tests determine students' degree of mastery of content after instruction and help teachers determine if skills have been mastered, or if review or reteaching may be needed.

Vmath Pre-Tests and Post-Tests were created using a detailed description of the content taught in each module to establish content validity. Content validity was established as follows by demonstrating that the test items are a sample of the content area in which the student has been instructed. First, the Pre-Tests and Post-Tests for each module were evaluated for validity by the content to see if each test problem was a good translation of the concepts taught within the module. Each question was then reviewed and identified as appropriate for the measure. Next, carefully selected experts in math instruction reviewed the tests to confirm that the problem selections were a good measure of mastery of the math content taught within each module. If any discrepancies were found by the experts, these problems were reviewed and replaced until agreement was reached.

Ongoing Assessment

In Vmath, teachers observe student performance and monitor student responses using detailed teaching dialogue, appropriate questioning techniques, and specific correction procedures. In the Getting Started section, the teacher monitors understanding of preskills and new skills. In the Guided Practice section, the teacher gauges student readiness to move to independent practice and uses correction procedures as

needed. In the Independent Practice section, the teacher checks students' daily work and uses correction procedures as needed. Students also monitor their own performance using the Independent Practice Progress Chart.

Test Prep and Error Analysis as Formative Assessment

The Test Prep and Error Analysis section of each lesson provides the teacher with a daily opportunity to analyze the errors made by each student, determine the exact cause of the error, and apply appropriate correction procedures to remediate and eliminate the error.

Online Data Management System

Teachers can efficiently monitor student progress when they administer assessments using the online data management system for Vmath called VPORT™. VPORT employs a three-step process in gathering, managing, and reporting data, and gives immediate feedback on each student's Benchmark and Progress Monitoring scores as well as Pre-Test and Post-Test results.

Utilizing VPORT, the teacher inputs scores from assessments and can then view each student's scores, compare class and individual progress between benchmarks, and print reports for multiple purposes. Teachers using the class report can review goal scores for each assessment and make informed instructional decisions and grouping decisions. VPORT allows teachers to identify students who may require additional support, keep track of progress, and make appropriate data-based instructional decisions that will allow students to experience success in math.

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